Tutorials 2

Filip Mazowiecki, Toghrul Karimov

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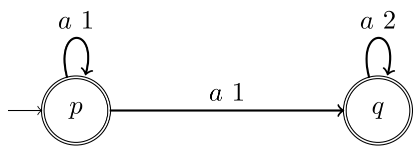
A deterministic weighted automaton is a weighted automaton such that the underlying finite automaton is deterministic. That is: there is one initial state, and for every state p and letter a there is exactly one transition (p, a, s, q). We say that two automata $\mathcal{A}$ and $\mathcal{B}$ are equivalent if $[\![\mathcal{A}]\!](w) = [\![\mathcal{B}]\!](w)$ for every word w.

1 Exercise

Consider the following automaton $\mathcal{A}$ over $(\mathbb{Q}, +, \cdot, 0, 1)$. The alphabet has only one letter $\Sigma = \{a\}$. The initial values are $I(p) = 1$ and $I(q) = 0$ and the final values are $F(p) = F(q) = 1$.



What is the value $[\![\mathcal{A}]\!](a^n)$? Define a deterministic automaton $\mathcal{B}$ over $(\mathbb{Q}, +, \cdot, 0, 1)$, which is equivalent to $\mathcal{A}$.

2 Exercise

Show (and prove) an example of a weighted automaton $\mathcal{A}$ over $(\mathbb{Q}, +, \cdot, 0, 1)$ such that there is no deterministic weighted automaton $\mathcal{B}$ over $(\mathbb{Q}, +, \cdot, 0, 1)$ which is equivalent to $\mathcal{A}$.

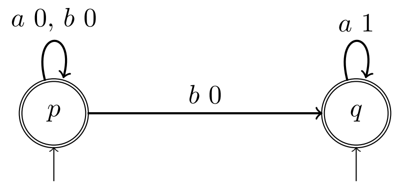
3 Exercise

Consider the example automaton $\mathcal{A}$ over the semiring $(\mathbb{N}_{-\infty}, \max, +, -\infty, 0)$ where the initial and final values of both states are 0. $\mathcal{A}$ computes the maximum of number of a's and number of b's.

Notice that $\mathcal{A}$ is nondeterministic because there are two inital states. Prove that there is no deterministic weighted automaton $\mathcal{B}$ over $(\mathbb{N}_{-\infty}, \max, +, -\infty, 0)$, which is equivalent to $\mathcal{A}$.

4 Exercise

Let $\Sigma = \{a, b\}$. Consider the weighted automaton $\mathcal{A}$ over $(\mathbb{N}_{-\infty}, \max, +, -\infty, 0)$ below. The initial function is $I(p) = I(q) = 0$ and the final function is $F(p) = F(q) = 0$.



Define a weighted automaton over the semiring $(\mathbb{Q}, +, \cdot, 0, 1)$ such that $[\![\mathcal{A}]\!](w) = [\![\mathcal{B}]\!](w)$.