Tutorials 4

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May 22, 2020

1 Exercise

This is more or less the lemma left to prove in Lecture 3.

- Define a family of finitely ambiguous automata $\mathcal{A}_n$ with the set of states $|Q| = n$ such that the ambiguity of the automaton is exponential in n.
- Show that if an automaton is finitely ambiguous then it is k-ambiguous for $k = 2^{\mathcal{O}(|Q|)}$.

2 Exercise

Recall the two criteria.

- $\mathcal{A}$ is not finitely ambiguous if and only if there are two states $p \neq q \in Q$ and a word w s.t. $p \xrightarrow{w} p$, $p \xrightarrow{w} q$ and $q \xrightarrow{w} q$.
- $\mathcal{A}$ is not polynomially ambiguous if and only if there is a state $p \in Q$ and a word w s.t. there are two runs $p \xrightarrow{w} p$.

Prove that both criteria remain true if we assume that the length of w is bounded by $p(|Q|)$ for some polynomial p.

Prove that this implies that given an automaton $\mathcal{A}$ both decision problems: if $\mathcal{A}$ is finitely ambiguous, and if $\mathcal{A}$ is polynomially ambiguous are in NLOGSPACE.

3 Exercise

Let $\mathcal{A}$ be an unambiguous weighted automaton over $(\mathbb{N}_{-\infty}, \max, +, -\infty, 0)$ such that $[\![\mathcal{A}]\!](w) \neq -\infty$ for every word w. Prove that there is a weighted automaton $\mathcal{B}$ over $(\mathbb{Q}, +, \cdot, 0, 1)$ such that $\mathcal{A}(w) = \mathcal{B}(w)$.

Note: this is a generalisation of Exercise 4 from Tutorials 2.