Tutorials 5

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1 Exercise

A formal power series of a sequence u_n is $U(x) = \sum_{i=0}^{\infty} u_n x^n$. For example for $u_n = 1$ we have $U(x) = \sum_{i=0}^{\infty} x^n = \frac{1}{1-x}$. Find the closed form of the formal power series of the following sequences:

- $a_n = n^2$;
- $u_n = \frac{1}{n}$ for $n > 0$ and $u_0 = 0$.

Hint. For the sequences considered in this course you can do basic operations on formal series and, for example, compute the derivatives (cf. Taylor series).

2 Exercise

Let $\lambda \in \mathbb{Q}$. Prove that for every $l, t \geqslant 0$,

$$\frac{1}{(1-\lambda x^l)^t} = \sum_{n=0}^{\infty} \binom{n+t-1}{t} \lambda^n x^{ln}$$

and that the sequence defined by this formal power series is definable by a polynomially ambiguous weighted automaton over $(\mathbb{Q}, +, \cdot, 0, 1)$.

3 Exercise

Consider the semiring $(\mathbb{Q}, +, \cdot, 0, 1)$ and a 1-letter alphabet. Show (and prove) an example of a sequence definable by a polynomially ambiguous weighted automaton but not definable by finitely ambiguous weighted automata.

4 Exercise

Prove that over 1-letter alphabet and over the semiring $(\mathbb{N}_{-\infty}, \max, +, -\infty, 0)$ for every finitely ambiguous weighted automaton there is an equivalent unambiguous weighted automaton.