Tutorials 6

Filip Mazowiecki, Toghrul Karimov

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1 Exercise

A probabilistic automaton with states Q alphabet Σ and transition function δ is a weighted automaton over the semiring $(\mathbb{Q}, +, \cdot, 0, 1)$ with the following restriction. For every $q \in Q$ and $a \in \Sigma$ the weights of transitions going out from q when reading a form a distribution:

- $s_{p,a,q} \in [0, 1]$
- $\sum_{p \in Q} s_{p,a,q} \leqslant 1$

where $s_{p,a,q}$ is the weight of each transition from q to p when reading a, with $s_{p,a,q} = 0$ if there is no such transition. One can see that then every word is assigned a probability, i.e. $[\![\mathcal{A}]\!](w) \in [0, 1]$. (Recall that we can always redirect the "missing probabilities" to a sink state, thus satisfying $\sum_{p \in Q} s_{p,a,q} \leqslant 1$ with equality.)

Prove that the problem (*) of given a probabilistic automaton $\mathcal{A}$, determining whether there exists a word w such that $[\![\mathcal{A}]\!](w) = \frac{1}{2}$ is undecidable. The goal is to modify the construction from the lecture that worked for automata over $(\mathbb{Q}, +, \cdot, 0, 1)$.

2 Exercise

One can observe that when $\frac{1}{2}$ is changed to any rational number $c \in (0, 1)$ then the decision problem (*) in the previous exercise remains undecidable. Prove that it is decidable for $c = 0$ and $c = 1$.

3 Exercise

Recall that the determinisation problem for a weighted automaton $\mathcal{A}$ is the question whether there exists a deterministic automaton $\mathcal{B}$ such that $\mathcal{A}$ and $\mathcal{B}$ are equivalent. Prove that over the semirings $(\mathbb{Q}, +, \cdot, 0, 1)$ and $(\mathbb{N}_{-\infty}, \max, +, -\infty, 0)$ if we assume that $\mathcal{A}$ is unambiguous, then the determinisation problem is decidable.

Note. Recall that these are big open problems when we don't assume anything about $\mathcal{A}$.