Eliminating recursion from monadic datalog on trees

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(joint work with Filip Murlak, Joanna Ochremiak and Adam Witkowski)

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Oxford

Introduction

(datalog: examples, problems, restrictions)

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Theorem (Chandra and Merlin)

Containment for CQ and UCQ is NP-complete.

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- **intensional** predicates ($buys$)
- one designated **goal** predicate ($buys$)

Datalog evaluation

UCQ with fixpoint

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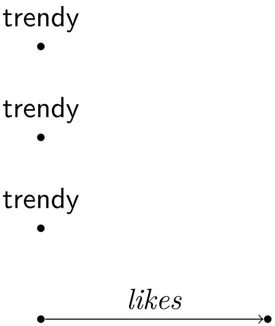

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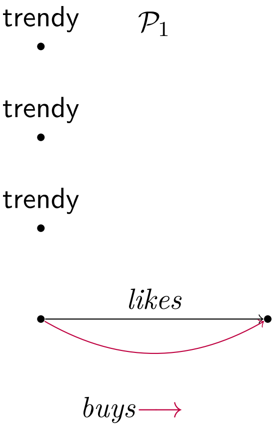

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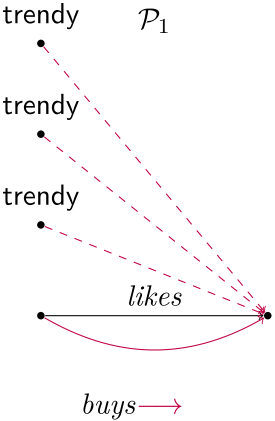

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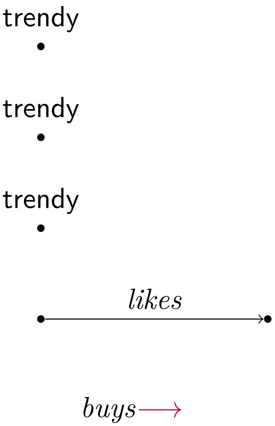

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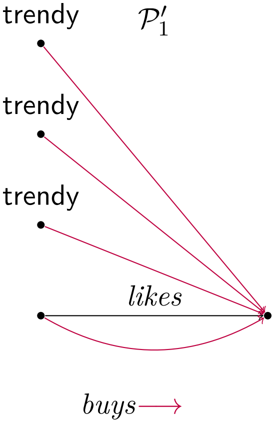

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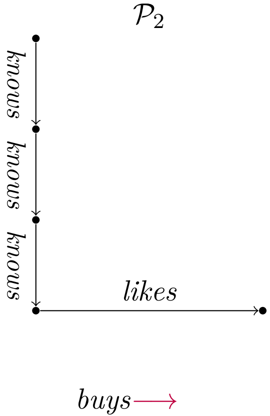

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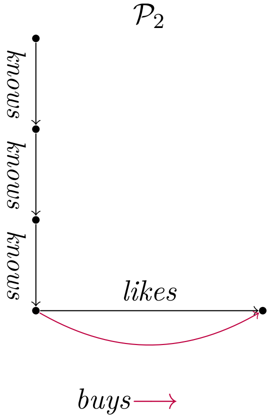

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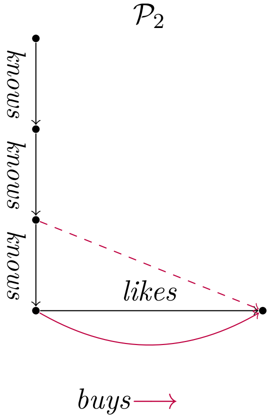

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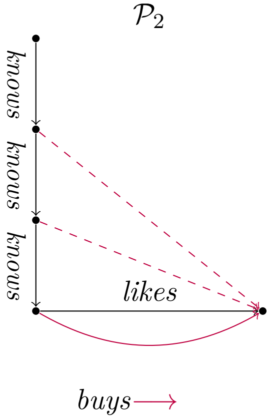

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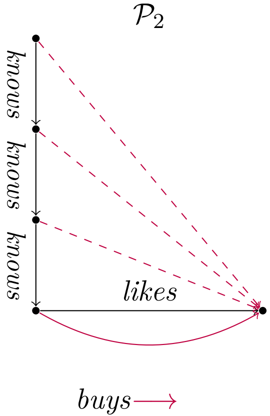

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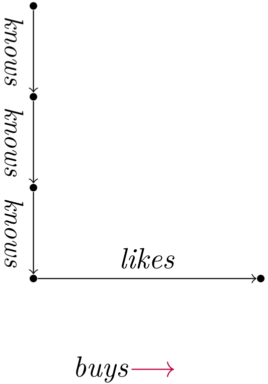

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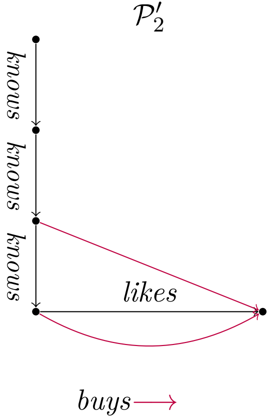

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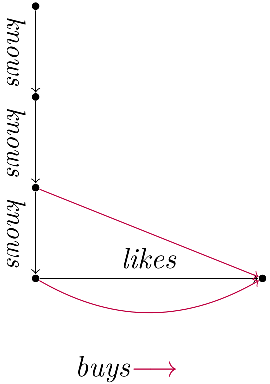

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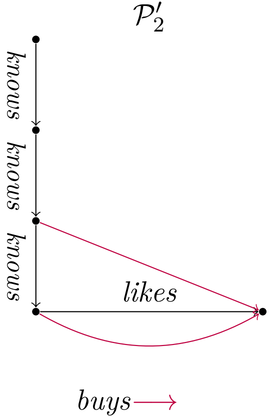

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Theorem (Chaudhuri,Vardi)

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A rule of thumb

Linearity does not change decidability, but lowers complexities.

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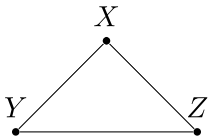

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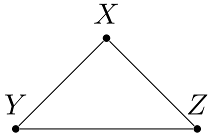

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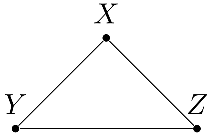

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A program is **downward** iff it is monadic

$+$ all G_r are trees with X in the root

Datalog on trees

(basic results on different models)

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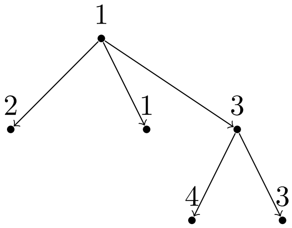

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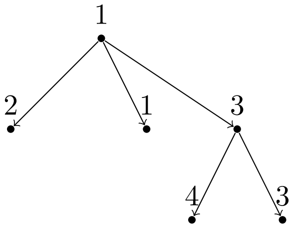

UCQ containment is decidable

Datalog on finite trees example

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\begin{aligned}
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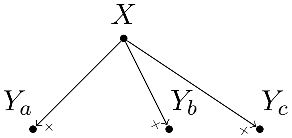

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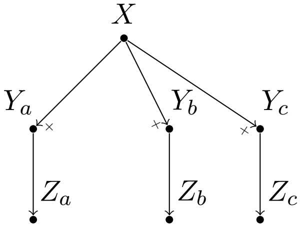
& \sim (X, Z_a), \sim (X, Z_b), \sim (X, Z_c)
\end{aligned}
$$



Datalog on finite trees example

$$
\begin{aligned}
P(X) \leftarrow\ & \downarrow_+(X, Y_a), \downarrow_+(X, Y_b), \downarrow_+(X, Y_c), \\
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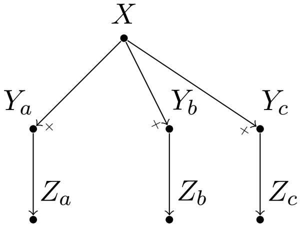

Not expressible in RegXPath

Datalog on finite trees

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Even satisfiability is undecidable [Abiteboul et al.]

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$+$ Some decidability results for bounded depth trees.

Our work

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- models: trees, where Σ is infinite

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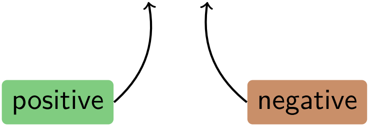

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negative

(possibly a UCQ)

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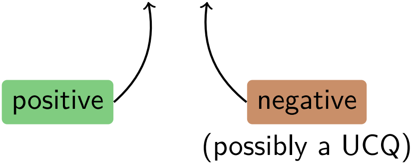

boundedness $\mathcal{P}$

Containment

(our results)

Containment in short

Containment of linear datalog in UCQ is undecidable.

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Trees		
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Canonical models: use an alphabet as big as possible

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Complexity 2EXPTIME (EXPSPACE for linear programs)

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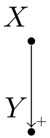

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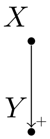

$X{\downarrow_+}Y, Y \sim X, \mathcal{P}(Y)$ $\qquad$ $X{\downarrow}Y, X{\downarrow}Z, \mathcal{P}(Y), Y \sim Z, \mathcal{R}(Z)$

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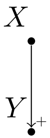

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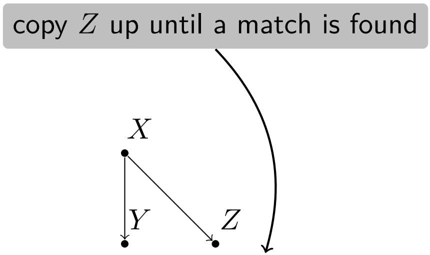

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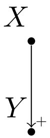

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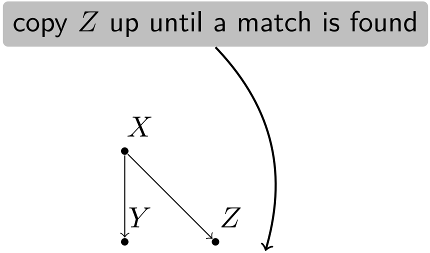

$X \downarrow Y, X \downarrow Z, \mathcal{P}(Y), Y \sim Z, \mathcal{R}(Z)$

Containment of child-only programs on unranked trees

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Containment of linear child-only programs is in 3EXPTIME,
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Proof similar as for downward programs

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Proof similar as for downward programs

Recently improved to 2EXPTIME-complete [Bojańczyk et al.]

Boundedness

(our results)

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On arbitrary models, or without descendant:

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We focus on the child-only fragment

Boundedness for child-only programs

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$\mathcal{P}$ is bounded iff every $\mathcal{P}(X)$ can be evaluated locally

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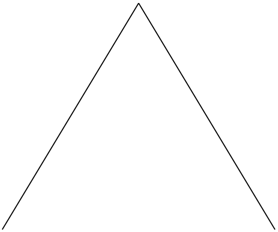

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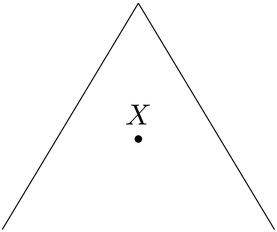

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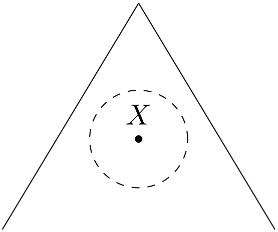

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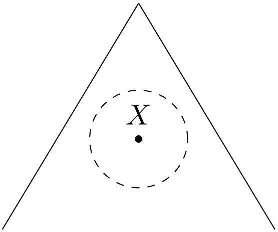

Using automata from containment we show a 2EXPTIME procedure

Boundedness for child-only programs (continued)

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Problematic example

Boundedness for child-only programs (continued)

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$$\begin{aligned}
\mathcal{P}(X_2) \;&\leftarrow R{\downarrow}X_1, X_1{\downarrow}X_2,\\
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&\quad R{\downarrow}Y_1, Y_1{\downarrow}Y_2, a(Y_2)
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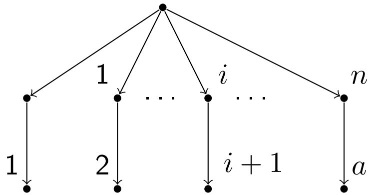

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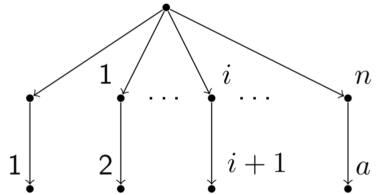

Previous key lemma does not work

Boundedness for child-only programs (continued)

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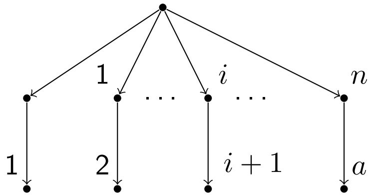
- Unranked trees

Problematic example

$$\begin{aligned}\mathcal{P}(X_2) &\leftarrow R{\downarrow}X_1, X_1{\downarrow}X_2,\\ &\quad R{\downarrow}Y_1, Y_1{\downarrow}Y_2, X_2 \sim Y_1\\ \mathcal{P}(X_2) &\leftarrow R{\downarrow}X_1, X_1{\downarrow}X_2,\\ &\quad R{\downarrow}Y_1, Y_1{\downarrow}Y_2, a(Y_2)\end{aligned}$$



Previous key lemma does not work

Work in progress . . .

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Is boundedness and effective boundedness the same problem?

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Forget about trees, restrictions, etc.

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